Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number | Candidate Number

Sample Assessment Materials for first teaching September 2018

(Time: 1 hour 30 minutes) | Paper Reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics FP3

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over

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Answer ALL questions. Write your answers in the spaces provided.

1. The curve C has equation

$$y = 9\cosh x + 3\sinh x + 7x$$

Use differentiation to find the exact x coordinate of the stationary point of C, giving your answer as a natural logarithm.

(6)

① $\frac{dy}{dx} = 9\sinh x + 3\cosh x + 7.$

$\frac{dy}{dx} = 0$

$9\sinh x + 3\cosh x + 7 = 0.$

$\therefore \frac{9e^{x} - 9e^{-x} + 3e^{x} + 3e^{-x}}{2} = 0 - 7.$

$\therefore 12e^{x} - 6e^{-x} = -14$

Multiply by e^{x}

$= 12e^{2x} + 14e^{x} - 6 = 0.$

$6e^{2x} + 7e^{x} - 3 = 0$

$(3e^{x} - 1)(2e^{x} + 3) = 0$

$e^{x} = 1/3.$

$x = \ln(1/3)$

$e^{x} = -2/3$ can't be used as it is negative and there are no real roots for ln(-ve no.)

Leave blank

2. An ellipse has equation

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta, 2\sin\theta)$, $0 < \theta < \frac{\pi}{2}$

The line L is a normal to the ellipse at the point P.

(a) Show that an equation for L is

$$5x\sin\theta - 2y\cos\theta = 21\sin\theta\cos\theta$$

(5)

Given that the line L crosses the y-axis at the point Q and that M is the midpoint of PQ,

(b) find the exact area of triangle OPM, where O is the origin, giving your answer as a multiple of $\sin 2\theta$

(6)

(a) At P, $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$.

$= \frac{2\cos\theta}{-5\sin\theta}$

$\therefore$ gradient of normal

$= \frac{5\sin\theta}{2\cos\theta}$.

$y - y_1 = m(x - x_1)$

$y - 2\sin\theta = \frac{5\sin\theta}{2\cos\theta}(x - 5\cos\theta)$

Multiply by $2\cos\theta$.

$2y\cos\theta - 4\sin\theta\cos\theta = 5x\sin\theta - 25\sin\theta\cos\theta$

$\therefore 5x\sin\theta - 2y\cos\theta = 25\sin\theta\cos\theta - 4\sin\theta\cos\theta$.

$\therefore 5x\sin\theta - 2y\cos\theta = 21\sin\theta\cos\theta$

as req.

(b) @ Q. $x = 0$

$-2y\cos\theta = 21\sin\theta\cos\theta$

$y_Q = \frac{-21\sin\theta}{2}$.

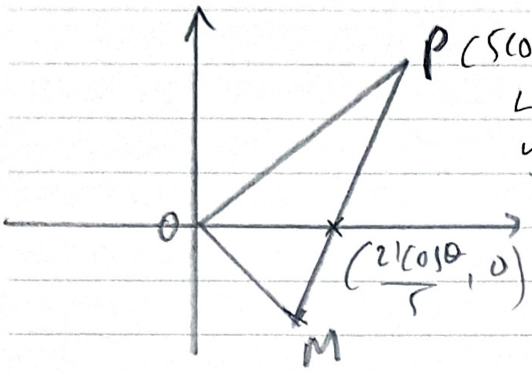
Q: $(0, -\frac{21}{2}\sin\theta)$ P: $(5\cos\theta, 2\sin\theta)$

$\therefore$ M has co-ordinates $x_m = \frac{5\cos\theta}{2}$

$y_m = \frac{2\sin\theta - 21/2\sin\theta}{2} = \frac{-17}{4}\sin\theta$.

Leave blank

Question 2 continued



L_1

$y = 0 \Rightarrow 5x\sin\theta = 21\sin\theta\cos\theta$

$\therefore x = \frac{21\cos\theta}{5}$

Area $= \Delta OPX + \Delta OMX$

Area $= \left|\frac{1}{2}\left(\frac{21\cos\theta}{5}\right)(2\sin\theta)\right| + \left|\frac{1}{2}\left(\frac{21\cos\theta}{5}\right)\left(\frac{-17}{4}\sin\theta\right)\right|$

$= \frac{21}{5}\sin\theta\cos\theta + \frac{357}{40}\sin\theta\cos\theta = \frac{105}{8}\sin\theta\cos\theta$

$= \frac{105}{16}\sin 2\theta$

Leave blank

3. Without using a calculator, find

(a) $\displaystyle\int_{-2}^{1} \frac{1}{x^2 + 4x + 13}\,dx$, giving your answer as a multiple of π, **(5)**

(b) $\displaystyle\int_{-1}^{4} \frac{1}{\sqrt{4x^2 - 12x + 34}}\,dx$, giving your answer in the form $p\ln\left(q + r\sqrt{2}\right)$,

where p, q and r are rational numbers to be found. **(7)**

(a) $\displaystyle\int_{-2}^{1} \frac{1}{x^2+4x+13}\,dx$

$= \displaystyle\int_{-2}^{1} \frac{1}{(x+2)^2+9}\,dx$

$= ~~\frac{1}{3}\arctan(1) - \frac{1}{3}~~$

$\left[\frac{1}{3}\arctan\left(\frac{x+2}{3}\right)\right]_{-2}^{1}$

$= \frac{1}{3}\arctan(1) - \frac{1}{3}\arctan(0)$

$= \frac{\pi}{12} \rightarrow \frac{1}{3}\arctan(1)$

(b) $4x^2-12x+34$

$= 4(x^2-3x+17/2)$

$= 4\left[\left(x-3/2\right)^2+\frac{25}{4}\right]$

$= \displaystyle\int_{-1}^{4} \frac{1}{\sqrt{4\left[(x-3/2)^2+\frac{25}{4}\right]}}\,dx$

$= \displaystyle\frac{1}{2}\int_{-1}^{4} \frac{1}{\sqrt{(x-3/2)^2+\frac{25}{4}}}\,dx$

$= \frac{1}{2}\left[\operatorname{arsinh} \frac{x-3/2}{5/2}\right]_{-1}^{4}$

$= \frac{1}{2}\left[\operatorname{arsinh} \frac{2x-3}{5}\right]_{-1}^{4}$

$= \frac{1}{2}\operatorname{arsinh}(1) - \frac{1}{2}\operatorname{arsinh}(-1)$

$= \frac{1}{2}\ln(1+\sqrt{2}) - \frac{1}{2}\ln(-1+\sqrt{2})$

$= \frac{1}{2}\ln(3+2\sqrt{2})$

4.

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ -1 & 1 & 1 \\ 1 & k & 3 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) Find $\mathbf{M}^{-1}$ in terms of k. **(5)**

Hence, given that $k = 0$

(b) find the matrix $\mathbf{N}$ such that

$$\mathbf{MN} = \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix}$$

(4)

(a) $\det(M) = \begin{vmatrix} 1 & 1 \\ k & 3 \end{vmatrix} - k\begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix}$

$= 3-k-k(-4) = 3+3k$

$$\begin{pmatrix} 3-k & -4 & -k-1 \\ 3k & 3 & 0 \\ k & 1 & k+1 \end{pmatrix}$$

apply $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}^T$

$$\Rightarrow \begin{pmatrix} 3-k & 4 & -k-1 \\ -3k & 3 & 0 \\ k & -1 & k+1 \end{pmatrix}$$

Then transpose matrix of minors

$\therefore M^{-1}$.

$$M^{-1} = \frac{1}{3k+3}\begin{pmatrix} 3-k & -3k & k \\ 4 & 3 & -1 \\ -k-1 & 0 & 1+k \end{pmatrix}$$

(b) $k=0$ $M^{-1} = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 4 & 3 & -1 \\ -1 & 0 & 1 \end{pmatrix}$

$M^{-1}MN = N$.

$$N = \frac{1}{3}\begin{pmatrix} 3 & 0 & 0 \\ 4 & 3 & -1 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix} 9 & 15 & 18 \\ 21 & 15 & 30 \\ 0 & -3 & -9 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 5 & 6 \\ 7 & 5 & 10 \\ 0 & -1 & -3 \end{pmatrix}$$

9

5. Given that $y = \text{artanh}(\cos x)$

(a) show that

$$\frac{dy}{dx} = -\operatorname{cosec} x$$

(2)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{6}} \cos x \, \text{artanh}(\cos x)\, dx$$

giving your answer in the form $a\ln\left(b + c\sqrt{3}\right) + d\pi$, where a, b, c and d are rational numbers to be found.

(5)

a) $y = \text{artanh}(\cos x)$

$\therefore \tanh y = \cos x$.

$\therefore \frac{dy}{dx} \operatorname{sech}^2 y = -\sin x$.

$\operatorname{sech}^2 y = 1 - \tanh^2 y$.

$\frac{dy}{dx} = \frac{-\sin x}{1-\tanh^2 y}$

$\therefore \frac{dy}{dx} = \frac{-\sin x}{1-\cos^2 x} = -\frac{\sin x}{\sin^2 x}$

$= -\frac{1}{\sin x} = -\operatorname{cosec} x$ as req.

(b) let $u = \text{artanh}(\cos x)$

$u' = -\operatorname{cosec} x$.

let $v' = \cos x \quad v = \sin x$.

$\int_0^{\pi/6} \cos x \,\text{artanh}(\cos x)\, dx$.

using integration by parts;

$\left[\sin x \,\text{artanh}(\cos x)\right]_0^{\pi/6}$

$+ \int_0^{\pi/6} 1 \, dx$.

$\frac{1}{2}\text{artanh}\left(\frac{\sqrt{3}}{2}\right) + [x]_0^{\pi/6}$

$= \frac{1}{4}\ln(7+4\sqrt{3}) + \frac{\pi}{6}$.

Leave blank

6. The coordinates of the points A, B and C relative to a fixed origin O are $(1, 2, 3)$, $(-1, 3, 4)$ and $(2, 1, 6)$ respectively. The plane Π contains the points A, B and C.

(a) Find a cartesian equation of the plane Π.

(5)

The point D has coordinates $(k, 4, 14)$ where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

(a) $\overrightarrow{AB} = \begin{pmatrix}-1\\3\\4\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix}$

$= \begin{pmatrix}-2\\1\\1\end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix}2\\1\\6\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}1\\-1\\3\end{pmatrix}$

$\therefore \begin{pmatrix}-2\\1\\1\end{pmatrix} \times \begin{pmatrix}1\\-1\\3\end{pmatrix} =$

$\begin{matrix}-2 & 1 & 1 & -2 & 1 & 1\\ 1 & -1 & 3 & 1 & 4 & 3\end{matrix}$

$\underset{\sim}{n} = \begin{pmatrix}4\\7\\1\end{pmatrix}$

$\underset{\sim}{r} \cdot \underset{\sim}{n} = \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}4\\7\\1\end{pmatrix}$

$= \underline{21}$

$\therefore 4x + 7y + z = 21$

(b) $\frac{1}{6}\left|\left(\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})\right)\right| = 6$

$\therefore \left|\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})\right| = 36$

$\overrightarrow{AD} = \begin{pmatrix}k\\4\\14\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}k-1\\2\\11\end{pmatrix}$

$\begin{pmatrix}k-1\\2\\11\end{pmatrix} \cdot \begin{pmatrix}4\\7\\1\end{pmatrix} = 36.$

$\Rightarrow 4k - 4 + 14 + 11 = 36.$

$4k = 15$

$\underline{\underline{k = 15/4}}$

Leave blank

7. The curve C has parametric equations

$$x = 3t^4, \quad y = 4t^3, \qquad 0 \leqslant t \leqslant 1$$

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is S.

(a) Show that

$$S = k\pi\int_0^1 t^5(t^2+1)^{\frac{1}{2}}\,\mathrm{d}t$$

where k is a constant to be found.

(4)

(b) Use the substitution $u^2 = t^2 + 1$ to find the value of S, giving your answer in the form $p\pi\left(11\sqrt{2} - 4\right)$ where p is a rational number to be found.

(7)

(a) $S = 2\pi\int_0^1 y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$

$= 2\pi\int_0^1 4t^3\sqrt{(12t^3)^2+(12t^2)^2}\,dt$

$= 2\pi\int_0^1 4t^3\sqrt{144t^6+144t^4}\,dt$

$= 2\pi\int_0^1 4t^3\sqrt{144t^4(t^2+1)}\,dt$

$= 2\pi\int_0^1 48t^5(t^2+1)^{1/2}\,dt$

$= 96\pi\int_0^1 t^5(t^2+1)^{1/2}\,dt$

$\therefore \underline{k = 96}$

(b) $u^2 = t^2+1$

$\therefore 2u\frac{du}{dt} = 2t \Rightarrow \frac{du}{dt} = \frac{t}{u}$

$\therefore dt = \frac{u}{t}\,du = \frac{u}{\sqrt{u^2-1}}\,du$

$t=1 \Rightarrow u^2 = 2 \Rightarrow u = \sqrt{2}$
$t=0 \Rightarrow u^2 = 1 \Rightarrow u = 1$
$\int_0^1$ becomes $\int_1^{\sqrt{2}}$

$t^5 = (u^2-1)^{5/2}$

$\therefore S = 96\pi\int_0^1 t^5(t^2+1)^{1/2}\,dt$

$= 96\pi\int_1^{\sqrt{2}}(u^2-1)^{5/2}\,u\cdot\frac{u}{\sqrt{u^2-1}}\,du$

$= 96\pi\int_1^{\sqrt{2}} u^2(u^2-1)^2\,du$

Leave blank

Question 7 continued

$$= 96\pi \int_1^{\sqrt{2}} u^2(u^4 - 2u^2 + 1)\, du.$$

$$= 96\pi \int_1^{\sqrt{2}} u^6 - 2u^4 + u^2\, du$$

$$= 96\pi \left[\frac{1}{7}u^7 - \frac{2u^5}{5} + \frac{1}{3}u^3 \right]_1^{\sqrt{2}}$$

$$= 96\pi \left[u^3\left(\frac{1}{7}u^4 - \frac{2}{5}u^2 + \frac{1}{3} \right) \right]_1^{\sqrt{2}}$$

$$= 96\pi \left[2\sqrt{2}\left(\frac{4}{7} - \frac{4}{5} + \frac{1}{3} \right) - \frac{8}{105} \right]$$

$$= 96\pi \left(\frac{22}{105}\sqrt{2} - \frac{8}{105} \right)$$

$$= 96\pi \times 2 \times \left(\frac{11\sqrt{2} - 4}{105} \right)$$

$$= \frac{64}{35}\pi (11\sqrt{2} - 4)$$

$$p = \frac{64}{35}$$

Leave blank

8. $$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 1$

$$I_n = I_{n-1} - \frac{1}{2n-1}\left(\frac{3}{5}\right)^{2n-1}$$

(5)

(b) Hence show that

$$\int_0^{\ln 2} \tanh^4 x \, dx = p + \ln 2$$

where p is a rational number to be found.

(5)

(a) $I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx$

$= \int_0^{\ln 2} \tanh^{2n-2} x \tan^2 x \, dx.$

$= \int_0^{\ln 2} \tanh^{2n-2} x \, ((1-\operatorname{sech}^2 x) dx$

$= \int_0^{\ln 2} \tanh^{2n-2} x - \operatorname{sech}^2 x \tanh^{2n-2} x \, dx.$

$= \int_0^{\ln 2} \tanh^{2n-2} x \, dx - \int_0^{\ln 2} \sec^2 x (\tanh x)^{2n-2} \, dx.$

$= \int_0^{\ln 2} \tanh x^{2(n-1)} \, dx - \left[\frac{(\tanh x)^{2n-1}}{2n-1}\right]_0^{\ln 2}$

$= I_{n-1} - \left[\frac{\tanh((\ln 2))^{2n-1}}{2n-1} - \frac{\tanh(0)^{2n-1}}{2n-1}\right]$

$= I_{n-1} - \left[\frac{(3/5)^{2n-1}}{2n-1} - 0\right]$

$= I_n = I_{n-1} - \frac{(3/5)^{2n-1}}{2n-1}$

as req.

Leave blank

Question 8 continued

b) $I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$.

$I_1 = I_0 - \frac{1}{1}(3/5)$

$\therefore I_1 = I_0 - \frac{3}{5}$.

$I_0 = \int_0^{\ln 2} \tanh^0 x \, dx = \int_0^{\ln 2} 1 \, dx$

$= \ln 2$

$\therefore I_1 = \ln 2 - 3/5$

$I_2 = \ln 2 - \frac{3}{5} - \frac{1}{3}\left(\frac{3}{5}\right)^3$

$= -\frac{84}{125} + \ln 2$